

# Measurement-Based Probabilistic Timing Analysis for Multi-path Programs

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**Abstract**—The rigorous application of static timing analysis requires a large and costly amount of detail knowledge on the hardware and software components of the system. Probabilistic Timing Analysis has potential for reducing the weight of that demand. In this paper, we present a sound measurement-based probabilistic timing analysis technique based on Extreme Value Theory. In all the experiments made as part of this work, the timing bounds determined by our technique were less than 15% pessimistic in comparison with the tightest possible bounds obtainable with any probabilistic timing analysis technique. As a point of interest to industrial users, our technique also requires a comparatively low number of measurement runs of the program under analysis; less than 650 runs were needed for the benchmarks presented in this paper.

**Keywords**—timing analysis, probabilistic real-time systems

## I. INTRODUCTION

The high-assurance segment of the embedded systems industry is compelled to continually increase the functionality and performance of its products. This can only realistically be achieved by adding complex performance-enhancing features to the system architecture in a competing drive to minimise cost and power consumption. However this is at odds with the state-of-the-art approaches to determining the timing behaviour of safety-critical systems.

Advanced hardware features, e.g. multiple cores and several cache levels, break the deterministic interpretation of system operation. While the behaviour of individual components is deterministic, the combination of possible hardware states makes precise modelling of worst-case behaviour difficult. Static timing analysis techniques are intrinsically limited by the complexity of constructing a sufficiently accurate model of the hardware, with consequences on the precision of a WCET estimate if the knowledge of state is incomplete. As the model becomes more detailed, the cost of computing tight upper bounds becomes prohibitive, and in some cases infeasible.

End-to-end measurement-based analysis relies on extensive testing performed on the real system. High-coverage input data, designed to stress the system, is used during testing to record the longest observed execution time (a ‘high watermark’). An engineering safety margin is then added to make allowances for the unknown. However, reasoning about the validity of the margin is extremely difficult – if possible at all – especially when the system may exhibit discontinuous changes in timing due to pathological cache access patterns, untested paths, or other unanticipated timing behaviour.

The cost of acquiring the required knowledge to perform trustworthy analysis can be significantly reduced by adopt-

ing a hardware/software architecture where execution timing behaviour minimises – or even eliminates – dependence on execution history, which is a factor affecting the internal hardware state. Several approaches are possible, and in this paper we achieve this independence by introducing time-randomisation to the execution resources that contribute to timing variability [6]. We couple this with new Probabilistic Timing Analysis (PTA) techniques. To some extent using ‘probabilistic’ bounds in high-integrity systems may seem counter-intuitive. However, the reality is that probabilistic modelling is a close match to the nature of the system. Let us consider an aircraft with a computing system on which PTA is to be applied. The mechanical parts in the aircraft are designed with a probability of failure in mind. Effects such as radiation, repeated physical stress and extreme temperatures introduce a low, but not null probability of failure for the computing hardware itself. Hence, the system as a whole has a distinct probability of failure. In that sense, timing failures can be considered just another type of failure that the system may experience. The objective of probabilistic timing analysis is to provide bounds to the execution time behaviour of software programs of interest, attached to acceptance thresholds similar in nature to the probabilistic failure rates required for certified systems (e.g.  $10^{-9}$  per hour of operation in airborne applications [2]). More precisely, for a software program on a given platform and a specified acceptance threshold, probabilistic timing analysis provides a probability distribution that bounds the probability that the execution time of that program may exceed a given value. We call this bound a probabilistic WCET (pWCET). The user of this technique is interested in the exceedance value that has a probability of occurrence no greater than the required acceptance threshold.

**Analysis approach:** So far, two main approaches to PTA have been proposed. Static PTA (SPTA) derives a-priori probabilities from a model of the system [7]. While reducing the amount of information needed from the platform and program to perform the analysis, this variant still requires a significant amount of information about the internal behaviour of execution resources. The second variant of PTA, Measurement-Based PTA (MBPTA), requires much less information, which makes it more attractive for industry. MBPTA derives probabilities for execution times by collecting observations of end-to-end runs of an application running on the target hardware. While a description of an SPTA technique was given in [7], a MBPTA technique is still missing. Some efforts have been made in using Extreme Value Theory (EVT) for the problem

of WCET estimation, but these methods are not based on appropriate assumptions about the statistical properties of the hardware as shown in [16], and are not obviously applicable by industry due to the simplified scenarios used in those works.

In this paper, we present for the first time a MBPTA technique based on a sound application of EVT that allows the analysis of multi-path, hence realistic, programs. We identify the requirements that this technique imposes on the experimental set-up and target platform so that the pWCET estimates we obtain can be applied with high confidence as an upper bound on execution time, and hence can be used in hard real-time systems. We pay special attention to the number of observations required by the user in order to obtain statistical significance, and for EVT to provide high-quality pWCET estimates. Our results show that our EVT-based MBPTA provides pWCET estimates with less than 15% pessimism with respect to Static PTA, which provides the tightest pWCET estimates that can be achieved with probabilistic timing analysis. The number of execution time observations required is feasibly low, 650 observations (runs) at most for all the benchmarks used in this paper.

**Paper organisation:** Section II introduces EVT. Section III details the requirements and how to apply EVT to single-path programs. Section IV extends the approach to multi-path programs. Section V presents our experimental setup and the results obtained. In Section VI we discuss the related work. In Section VII we state the main conclusions of this study.

## II. EXTREME VALUE THEORY FOR TIMING ANALYSIS

Let us assume a given processor and program for which we would like to provide pWCET estimates. Further assume that the execution of the program is observed 1,000 times and the execution times are collected. If those execution times can be modelled with independent and identically distributed (i.i.d.) random variables [19], then the pWCET of a program can be derived by collecting enough observations to construct an Empirical Cumulative Distribution Function, ECDF. From the inverse ECDF it is possible to compute an estimated pWCET as  $\overline{pWCET}$  for a desired probability. Such an approach has an important requirement that we must have a good model of the tail-end behaviour of the ECDF; however, we would need a vast number of observations to model these low-probability events. In this case, as the number of observations is 1,000, the maximum confidence is  $p(\text{execution time} > WCET_{\text{estimation}}) = 10^{-3}$  per run.

Alternatively, one may use mathematical methods to predict, out of a small enough number of samples, pWCET bounds for exceedance probabilities smaller than  $10^{-n}$ , where  $n$  is a required level of confidence. In this paper we explore a mathematical method based on Extreme Value Theory (EVT). This theory estimates the probability of occurrence of extreme values, whether high or low, which are known to be rare events [9]. More precisely, EVT predicts the distribution function for the maximal (or minimal) values of a set of  $n$  observations,

which are modelled with random variables. The EVT theory is analogous to Central Limit Theory [28] but instead of estimating the average, EVT estimates the extremes [15]. As in our work we are interested in the high values that bound the pWCET, we consider the EVT prediction for maximal values of a set of observations.

**Theorem 1.** [15] *Let  $\{\mathcal{X}_1, \mathcal{X}_2, \dots, \mathcal{X}_n\}$  be a sequence of i.i.d. random variables and let  $\mathcal{M}_n = \max\{\mathcal{X}_1, \mathcal{X}_2, \dots, \mathcal{X}_n\}$ . If  $F$  is a non degenerate distribution function and there exists a sequence of pairs of real numbers  $(a_n, b_n)$  such that  $a_n \geq 0$  and  $\lim_{n \rightarrow \infty} P(\frac{\mathcal{M}_n - b_n}{a_n} \leq x) = F(x)$ , then  $F$  belongs to either the Gumbel, the Frechet or the Weibull family.*

The main result of EVT is provided in Theorem 1, where  $F$  denotes the common distribution function of  $n$  random variables. In our approach, random variables are used to model the execution time of programs. In order to apply Theorem 1 two main hypotheses must be verified: that the  $n$  random variables are independent and identically distributed (see Section III-C) and the existence of the sequence of real numbers  $(a_n, b_n)$ .

The Gumbel, Frechet and Weibull are particular cases of the Generalised Extreme Value (GEV) distribution which has the following Cumulative Distribution Function (CDF):

$$F_{\xi}(x) = \begin{cases} e^{-(1+\xi \frac{x-\mu}{\sigma})^{\frac{1}{\xi}}} & \xi \neq 0 \\ e^{-e^{-\frac{x-\mu}{\sigma}}} & \xi = 0 \end{cases}$$

GEV is defined by three parameters: shape ( $\xi$ ), scale ( $\sigma$ ) and location ( $\mu$ ). By determining these three parameters, we prove the existence of the sequence of real numbers  $(a_n, b_n)$ . If the shape parameter  $\xi = 0$  then  $F_{\xi}$  is a Gumbel distribution, which is the distribution we use in this analysis.

When EVT is applied, we must determine its three parameters [14] and hence the appropriate CDF (Gumbel, Frechet or Weibull) is selected. To that end, a goodness-of-fit test is needed. However, not all tests are appropriate for fitting extreme values. For instance, in [18] the authors use the  $\chi^2$  test which is known to perform incorrectly for extreme values in any classical test [14]. One test that is proven correct when fitting the Gumbel CDF is the exponential tail (ET) test [14].

EVT has previously been applied to the WCET estimation problem in [9], [18]. The main limitations of these two papers, which are shown in [16], are the following: the assumption of independent and identically distributed random variables, and fitting a continuous distribution to discrete values. In Section III we correct the application of EVT to WCET estimation by proposing solutions to these limitations. Moreover, we also provide an alternative to the  $\chi^2$  test in [18] for fitting the Gumbel CDF. Finally, we show how to apply EVT to multi-path programs.

### A. Hypotheses

In this section we detail the hypotheses needed for a correct application of EVT.

**Independence and identical distribution:** The main hypothesis for Theorem 1 is that the sequence of random variables is independent and identically distributed. We provide here the definition of this notion. In following sections we show how to achieve this property when applying EVT to obtain pWCET estimates.

**Definition 1** (Independent Random Variables). *Two random variables  $\mathcal{X}$  and  $\mathcal{Y}$  are independent if they describe two events such that the occurrence of one event does not have any impact on the occurrence of the other event.*

**Definition 2** (Identically distributed Random Variables). *A sequence of random variables is identically distributed if all random variables have the same probability distribution.*

We say that a sequence of random variables is independent and identically distributed (i.i.d.) if they have the same distribution function and they are independent from one another. A sequence of (fair) die rolls where each roll is a random variable is i.i.d., since the random variables describe the same event and the outcomes are obtained from independent events (as the outcome of one roll is independent of the outcome of another roll).

**Selection of an EVT distribution:** In order to apply Theorem 1, parameters describing a valid EVT distribution must be chosen (Gumbel, Frechet or Weibull). This requires an appropriate statistical test. Previous applications of EVT to WCET estimation [9], [18] indicate that the Gumbel distribution fits well the problem of WCET estimation. We validate this hypothesis by checking whether a given ECDF fits the Gumbel CDF, for which we use the exponential tail (ET) test [14]. The ET test assumes that the data set is modelled by  $n$  i.i.d. random variables  $\mathcal{X}_1, \dots, \mathcal{X}_n$  described by the CDF  $F$ . The ET test compares this to the Gumbel CDF:  $\mathcal{H}_0 : F = F_\xi$ . If the test rejects the hypothesis  $\mathcal{H}_0$ , then the hypothesis  $\mathcal{H}_1 : F \neq F_\xi$  must be true. The details for applying ET to our data are provided in Section V.

### B. Steps in the application of EVT

There are two main steps in the application of EVT.

**Grouping:** The objective of this step is to collect, from the original distribution, the values which fit the tail, and hence can be modelled with the Gumbel distribution. The values used as input for EVT are grouped into *blocks* of equal length  $m$  by applying the *block maxima* method. The maximum value observed in each block constructs a new sample, the block maximum series.

The size of the blocks determines the portion of the original distribution that is considered the tail. The larger the block size, the better the fit the tail; however, increasing the block size results in fewer blocks and thus fewer values in the final sample. In the extreme case, if a single block is used, we will have only one block maximum value corresponding to the maximum of the original distribution. Conversely, if we use as many blocks as elements in the original distribution, the distribution of the block maximum values will gather the

entirety of the original distribution, rather than capturing the essence of the tail.

The theoretical basis of the block maxima method is built on the following theorem which says that if some data fit the Gumbel CDF, then the maxima of blocks of those data fit the same Gumbel CDF.

**Theorem 2.** [10] [Grouping] *Let  $\mathcal{X}$  be a random variable. If the  $n$  variables  $\mathcal{X}_1, \mathcal{X}_2, \dots, \mathcal{X}_n$  obtained from  $\mathcal{X}$  fit a GEV, then the maxima of any blocks obtained from  $\mathcal{X}$  fit the same GEV.*

**Distribution fitting:** If the ET test shows that our observations follow a Gumbel distribution, we next estimate the two remaining parameters,  $\mu$  and  $\sigma$  using the block maxima series. Their practical estimation is obtained by applying linear regression to the QQ-plot of our data [11]. A QQ-plot (quantile plot) is a plot of the empirical quantile values of observed data against the quantiles of the standard form of a target distribution. Given that the block maxima values follow a Gumbel distribution, then the points on the QQ-plot form a straight line [11]. The slope and the intercept of the best-fit line through these points can be used as estimators for the Gumbel  $\mu$  and  $\sigma$  parameters, respectively.

## III. EVT FOR SINGLE-PATH WCET

When applying EVT to the problem of providing pWCET estimates it is necessary to ensure that the assumptions made by EVT hold, so that the estimates obtained are sound. As we show in this section, these hypotheses have implications for the way experiments are run and the behaviour of the system. In particular, EVT assumes that the execution times of the system are independent between runs, and identically distributed.

We assume that the program for which a pWCET is to be estimated contains no system calls and that it is run in isolation. This is not a fully realistic scenario for production systems, but the assumptions match those used for current WCET analysis techniques; the interaction with other tasks and the operating system is taken into account during system integration. For this section, we further assume that the program contains a single-path of execution (in the next section we focus on multi-path programs).

In a fully deterministic system, a single execution path, when run from identical starting conditions, should have a single execution time. In practice this is not the case due to interference effects caused by hardware features such as DRAM refresh cycles, and in any case it is rare, or even impossible, to ensure that execution begins from identical starting conditions every time. Past approaches have attempted to use EVT to model this execution time variability, but there are no statistical guarantees about the nature of the variability. Rather than attempting to model such complex behaviour, our approach is to bring the behaviour closer to the model, thus ensuring that the variability in execution time conforms to

distributions which can be modelled with statistical methods such as EVT.

By ensuring that timing behaviour of each source of jitter is independent, the combination of the response times (latencies) incurred by the involved resources will converge on average to a closed form distribution. In most cases this is the normal distribution due to the finite variance of the input distributions, according to Central Limit Theorem. Sources which cannot be modelled by independent random variables must be considered as constants that always take their upper bound.

#### A. Input Data Dependence

For the single-path case, with known, fixed input data on a simple architecture, we can exactly compute the distribution of execution times which will be observed when the path is executed by using a static probabilistic model of the hardware behaviour [7]. This provides an excellent way to evaluate the behaviour of EVT when applied to a time-randomised architecture, and gives greater confidence that the method is still sound when applied to more complex architectures.

In order to perform measurement-based analysis, the user must provide a suite of input data to the program, designed to stress the program and produce worst-case behaviour. The choice of the data may affect the timing behaviour of the software, even if it does not affect the path taken. For example, dividing some numbers may take more cycles than other numbers, and data structures which contain pointers may refer to the same or different memory locations depending on the input, which affects the memory access pattern, and hence cache latencies.

Users cannot be asked to provide the ‘worst’ or even statistically representative data during the testing phase; they do not know (in general) what this data will be at deployment time, so we must design the platform in a way that causes no temporal effect to stem from input data.

There are hardware proposals which force operations, such as divisions, to always take their upper-bound execution time, which is a safe (pessimistic) assumption [23]. In [23] the authors introduce a ‘Worst-Case Mode’, that simply delays each request to a resource until its maximum execution time is observed. This is configurable by software and once active this technique is transparent to the executing programs. For the purposes of this paper such a technique has to be present in the target architecture. It will be activated during the collection of execution times and can be switched off during deployment.

A similar solution can be applied to data-dependent memory accesses which must be flagged so that effects due to the input data can be accounted for. Most current Instruction Set Architectures (ISA) include some hint bits in the operation code of memory operations, which are used, between others, to advise the processor whether to cache some data or to activate the data prefetcher. In the former case, if the designer detects that the data for a given load is going to have a single use, it activates this bit for the opcode of the load. Analo-

gously, in our case, input data dependent memory operations have to be identified in the object code and flag them by means of these hint bits to prevent cache or other performance-improving features being used for these instructions.

#### B. Platform Requirements

Our EVT-based MBPTA approach provides a pWCET estimate attached to an exceedance probability. Our approach to make this hold is to ensure that all sources of execution time variation in the system must either be safely upper-bounded or have a true probabilistic behaviour such that their measurements on target capture the outcome of the events that make the execution time vary.

Our approach to this end is to randomise the timing behaviour of the hardware resources with latency too high to upper-bound [7] and to upper-bound those whose incurred pessimism is acceptable, as in the case of the input data that we explained before. Once that is done, the only events that can affect the execution time are random. Consequently, the observed frequencies of each execution time represent probabilities with a confidence level captured by the EVT method. Hence the system behaviour observed during testing can be used to tell its behaviour during actual operation.

This approach cannot be applied to a standard not-time-randomised architecture as the frequencies of the execution times observed during testing do not represent actual probabilities. This is because the events that affect execution time are not randomised. Hence, no probabilistic guarantees can be determined from the observed behaviour on the probability of occurrence of given execution times in the future. A more detailed description of the hardware requirements can be found in [7].

#### C. Independent and Identically Distributed Observations

In order for the i.i.d. properties to hold in our case, the architecture we propose must ensure that each execution run is independent. For the purposes of this experiment, we will ensure that runs are independent by forcing the state of the hardware to be reset before each run begins.

Identical distribution at the path level requires only that the latencies which make up the total execution time are independently chosen each time a path is executed. Each possible execution time that a path can take must have an associated probability. Note that this does not require that the probability distribution for an instruction is independent of the sequence of preceding instructions, only that the observed timing behaviour in one run does not affect the timing behaviour of other execution runs.

The architecture we use for this study (see Section V-A) exhibits the properties required to sustain the application of EVT-based MBTA.

#### D. Continuous vs. Discrete Functions: minimum number of observations

In [16] it is shown that using the Gumbel distribution function, or any continuous function, to approximate a discrete

function of the execution time values produced by a program, could yield results that do not bound the execution time from above at all points. In [16] there are proposed different solutions to this problem by either the continuous function fitting the upper bound of the exceedance distribution function and thus overestimating the discrete function, or by adding an offset to the distribution which accounts for this effect. For instance for the single path we consider here, the maximum offset required is likely to be a small number of cycles.

In our paper we introduce an overestimation of the discrete function when applying EVT (Section II-B) through the block maxima method before the distribution fitting is applied. This produces a shift of the distribution, which makes the worst-case estimate pessimistic when compared to the ‘real’ distribution of execution times in our architecture. In order to obtain high confidence in such a distribution, we require that the number of observations is sufficient to produce an accurate description of the program’s execution time. We call this the **minimum number of observations**, and we define it as follows:

**Definition 3.** *For a given sequential program  $P$  and an architecture  $A$ ,  $n$  is the minimum number of observations if there does not exist  $m \in \mathbb{N}$  where  $m > n$  such that the Gumbel CDF obtained for  $n$  observations is an upper bound for the WCET of  $P$  and also exceeds the CDF obtained for  $m$  observations.*

We obtain the *minimum number of observations* by comparing the variation in the EVT tail projection as we consider an increasing number of runs. We follow an iterative process:

- 1) We start by running  $N_{current} + N_{delta}$  times the program under study on the target platform.
- 2) We then make two tail projections with EVT, one using  $N_{current}$  runs and the other using  $N_{current} + N_{delta}$ .
- 3) If the difference between the two EVT distributions is below a given *difference threshold* we stop the process. If it is over the threshold, we consider all previous runs as  $N_{current}$ , or in other words we make  $N_{current} = N_{current} + N_{delta}$ , and we make  $N_{delta}$  more runs, consider the part of the sample and repeat the process from step 2.

As we keep adding more runs to  $N_{current}$  the proportional effect that  $N_{delta}$  new runs has on the EVT reduces, so a convergence of the algorithm is ensured. On the other hand, there is no guarantee that there is a strict convergence so we can have some local minima. In order to take into account this non strict convergence, we change the algorithm so that instead of stopping the process as soon as the difference between the two EVT distributions is below a given difference threshold, we stop the process when this is the case in several consecutive iterations. For the benchmarks used in this paper, considering 5 consecutive iterations below the threshold is sufficient to deal with the hysteresis introduced by local minima.

The metric we use to compare the two EVT distributions

(functions) is called the continuous rank probability score defined as  $CRPS = \sum_{i=0}^{+\infty} [f_X(i) - f_Y(i)]^2$  [5]. This test is defined for any two distribution functions  $f_X$  and  $f_Y$  as long as they operate on the same value domain. In this paper we set as difference the threshold  $10^{-1}$ . This threshold has to be interpreted as the significance level  $\alpha$  in hypothesis tests. The lower its value the larger is the confidence on the result. It is up to the user of our technique to set a proper value to this difference threshold. However, note that the lower the threshold the higher the required number of observations.

### E. Step-by-step application of EVT

EVT is applied to single-path programs by following these steps for a number of rounds until a stable result is obtained:

1) *Collect observations* – The first task is to gather a number observations of the execution time of the program. We suggest starting with around 100 values. In each round, an additional  $N_{delta}$  observations are made and included in the data used for the next steps. This process is described in Section III-D.

2) *Grouping* – Block maxima sampling, described in Section II is used to convert the measured frequency distribution into a worst-case distribution suitable for application of EVT.

3) *Fitting* – The Gumbel distribution which best fits the observation is estimated. For this we use parameter estimation based on [14] as shown in Section II.A.

4) *Comparison* – If this is not the first round, the 1-CDF for the current round is compared to the result of the previous round using the CRPS metric.

5) *Convergence* – When the CRPS metric reports values below a given threshold (0.1 in the examples in this paper) for a number of consecutive rounds (5), we consider the distribution to have converged, and no more observations need to be collected.

If the distribution has not yet converged, the process starts another round at Step 1.

6) *Tail extension* – The final parameters for the Gumbel distribution are used to compute the execution time values for probabilities down to a lower bound. In our examples we use  $10^{-16}$  as this is the limit of precision for floating point numbers in our prototype tool. Greater precision can easily be obtained using an arbitrary-precision floating point implementation such as MPFR [12].

This process is used to produce the results in Section V-C.

## IV. EVT FOR MULTI-PATH WCET

### A. General considerations

The application of EVT MBPTA to multiple-path programs is straightforward in principle, with three important provisos: 1) the compound distribution resulting from multiple paths must provably exhibit the required i.i.d. properties; 2) a sufficient number of observations must be made for each path; and 3) the result only applies to the set of observed paths. In the following we discuss each of them in isolation.

### Independent and identically distributed observations

The i.i.d. property that already holds for single-path programs may be preserved for multi-path programs by either selecting inputs randomly when running the measurement runs and grouping them sequentially, or testing with all inputs and selecting results randomly, without replacement, when performing grouping. We assume that there is a direct and traceable correspondence between the input (data and state) to an observation and the path taken by the execution.

**Minimum Number of Observations.** EVT is insensitive to the relative frequencies of different paths in the measured data: the process of extracting the block maxima distribution allows the data corresponding to worst-case paths to dominate the result. There is however a requirement that each path must have been observed a minimum number of times to allow the behaviour to be sufficiently well characterised (see Section III-D). One possible way to ensure this property is to execute each input from the test set at least this minimum number of times.

**Path Coverage.** Extending this problem in another direction, the result is only valid for the paths which are observed. Claims on the pWCET for untested paths cannot be made, as the execution times for those paths would no longer be drawn from an identical distribution. This inherently couples the concept of i.i.d. with the path coverage achieved by the input data.

The issue of path coverage introduces a problem for the usefulness of the result, as full path coverage in complex applications is infeasible: the combination of loops and branches creates an explosion in the number of possible paths through the program. Therefore we must impose a caveat on the result obtained by EVT: that it is valid only for a set of ‘observed paths’, that is, those paths covered by the observations made.

In order to apply our method in a realistic scenario therefore, it will be necessary to strike a balance between precision and practicality. This may be achieved by limiting the scope of the program being considered for pWCET estimates. For instance, the units over which EVT may be applied could be defined as the largest partitions of the control-flow graph over which full path coverage has been observed. Once estimates have been calculated for these units, the results may be combined using a tree-based composition, as in [4]. This allows the testing burden to be reduced, at the expense of increasing pessimism in the overall estimate. To further improve applicability: (1) we recommend that the measurement runs are performed as part of functional testing, so that our method attains no less path coverage than that required for functional testing, at the added cost of achieving a minimum number of observations per path, which may be larger than that required by functional testing alone; (2) we note that the attained path coverage can be recorded with modest enhancement to the test environment and existing methods can be used to determine unobserved paths of interest, i.e., those that could cause longer execution time than that captured by the observations (e.g. [26]).

### B. Step-by-step application

The step-by-step application of EVT was described for single-path programs in Section III-E. The only change for multi-path programs is to Step 1, collection of observations.

With multiple paths, each path must have sufficient observations to characterise its behaviour. Therefore, the number of observations required is multiplied by the number of paths. This means that we should start with, for example  $100 \times P$  where  $P$  is the number of inputs, assuming that each input exercises a distinct path.

We should also increase the number of additional observations in each round to  $P \times N_{delta}$  so that the paths each have a sufficient number of observations to determine a change in the estimated distribution.

## V. EXPERIMENTAL EVALUATION

In this section we first present the experimental environment we used to evaluate our analysis technique. Next, we introduce SPTA as a reference probabilistic timing analysis method for single-path programs. Finally, we present the results that we obtained, in terms of pWCET estimates, for both single-path and multi-path programs.

### A. Experimental Set-up

We run all experiments on a modified version of SoCLib [1], an open platform for virtual prototyping of multi-processors system on chip (MP-SoC). We modelled a pipelined processor featuring data and instruction caches. We used 4Kilobyte, 1024-way, 4-byte line, fully-associative data and instruction caches deploying a random-replacement policy. The latency of each instruction is fixed, except for the first stage (fetch) in which the instruction cache is accessed. The latency of the fetch stage depends on whether the access hits or misses in the instruction cache: a hit has 1-cycle latency and a miss has 100-cycle latency. After the decode stage, memory operations access the data cache so they can last 1 or 100 cycles depending on whether they miss or not. Overall, the possible latencies of a non-memory operation are  $(N+1, N+100)$  depending whether it hits or not in the instruction cache, where  $N$  is the number of cycles it takes executing the instruction (e.g. integer additions take 1 cycle). Memory operations have 3 possible latencies: 2 cycles (1+1) when it hits both instruction and data caches; 200 (100+100) when it misses in both caches; 101 (1+100) when it hits only one of both caches.

**Benchmarks.** We used eight benchmarks of the EEMBC Autobench suite [24] as reference for the analysis of single-path programs. EEMBC Autobench is a well-known benchmark suite that reflects the current real-world demand of some embedded systems. We used: a2time (AT), aifirf (AI), cacheb (CA), canldr (CN), puwmod (PU), rspeed (RS), tblock (TB) and ttsprk (TT).

For multi-path program analysis we used several Mälardalen benchmarks [17], which are commonly used in the community to evaluate and compare different

types of WCET analysis tools and methods. In particular we used: bs (BS), cnt (CNT), compress (COM), crc (CRC), insertsort (INS), qsort (QSO) and select (SEL). For each of these programs we developed several input sets that exercise different paths.

We also designed a multi-path synthetic benchmark to better understand how the multi-path program time analysis works. In the synthetic benchmark we know (1) the different paths of the program, (2) the different input set that exercise each path and (3) the longest path. Thus we may compare the pWCET estimate we obtain with our single-path time analysis when applied to the worst-case path against the pWCET estimate we obtain with our multiple-path time analysis when applied to a set of paths which includes the worst-case path.

### B. SPTA

Static probabilistic time analysis, which was first introduced in [7], is used in this paper as a term of reference for our approach in this work since SPTA provides a very tight pWCET estimate to the actual distribution of execution times of the program. However, it does so at the cost of being extremely more onerous to execute since it operates at the level of individual processor instructions and also requires exact knowledge of the hardware model, much like classic static timing analysis.

SPTA in fact uses as input the possible latencies that every instruction may take and the probability associated with them. This information is usually represented in the form of an execution time profile. Listing 1 shows an example for a sequence of instructions, where  $i_h, i_m$  are the probability of hit and miss in the instruction cache respectively; and  $d_h, d_m$  are the probability of hit and miss in the data cache.

- 1:  $\{(2, 101, 200), (i_h0 \times d_h0, i_h0 \times d_m0 + i_m0 \times d_h0, i_m0 \times d_m0)\}$
- 2:  $\{(2, 101), (i_h1, i_m1)\}$
- 3:  $\{(2, 101, 200), (i_h2 \times d_h2, i_h2 \times d_m2 + i_m2 \times d_h2, i_m2 \times d_m2)\}$

Listing 1. Execution time profile for a sequence of three instructions. Instructions 1 and 3 access memory, whereas 2 does not.

Each line contains the execution time profile for one instruction: the first tuple are the latencies in the event each possible hit/miss combination occurs. As explained above the possible latencies of a non-memory operation are  $(X+1, X+100)$  depending on whether the instruction cache access is a hit or not. In the case of the second instruction in Listing 1,  $X$  – the inherent instruction latency – is equal to 1. Memory operations, i.e. instructions 1 and 3 in Listing 1, have three possible latencies:  $(2, 101, 200)$ . The numbers following this group are the probabilities of observing the corresponding latencies.

For fully-associative random-replacement caches such those used in this work, these probabilities cannot be computed using the formula used in [29]:  $P_{hit} = \left(\frac{N-1}{N}\right)^k$  where  $k$  is the reuse distance and  $N$  the number of lines in the

cache. In fact, this formula does not satisfy our independence requirement: given two consecutive accesses to an address  $A$ , the hit probability is lower for the second access if all the intermediate accesses were hits than if they were misses. This is due to conditional probability: if we know that address  $B$  was *not* evicted because we observe a hit, then the chance that we evicted  $A$  instead is higher than if we observe a miss. Determining the exact hit probability has exponential cost in the size of  $K$ ; however, a pessimistic lower bound on hit probability can be computed by applying equation 1. We assume that there is an upper bound on the amount of information which could be known about the cache contents over the reuse distance. We take the case where all of the addresses accessed are unique and these entries are known to be in the cache for all of the reuse distance, reducing the effective cache size available to  $A$  from  $N$  to  $N - (K - 1)$ . Therefore, independence is ensured and distributions obtained from this formula may be convolved.

$$P(hit) = \begin{cases} \left(\frac{N-(K-1)-1}{N-(K-1)}\right)^K & \text{if } K < N \\ 0 & \text{if } K \geq N \end{cases} \quad (1)$$

Based on this trace of execution time profiles, we start an iterative process in which we *convolve* them [7]. The convolution of two execution time profiles leads to a new one in which all possible pairs of execution times from the two execution time profiles are added and the probabilities multiplied. For example, let  $E_1 = \{(2, 101, 200), (0.1, 0.4, 0.5)\}$  and  $E_2 = \{(2, 101), (0.6, 0.4)\}$  be two execution time profiles.

Then their convolution is:

$$E_r = \{(2+2, 2+101, 101+2, 101+101, 200+2, 200+101), (0.1 \times 0.6, 0.1 \times 0.4, 0.4 \times 0.6, 0.4 \times 0.4, 0.5 \times 0.6, 0.5 \times 0.4)\}.$$

And given that  $101 + 101 = 2 + 200$  we collapse all duplicate pairs resulting in:

$$E_r = \{(4, 103, 202, 301), (0.06, 0.28, 0.46, 0.2)\}.$$

This process is repeated for all the instructions of the program. The final execution time profile is the complete execution time distribution of the program, including the worst-case behaviour in the tail. This can be used to make pWCET estimates by examining the exceedence function.

### C. Results for Single-Path Programs

In order to obtain sound pWCET estimates through the application of EVT, the platform/data under consideration has to meet certain statistical properties. One of the ways to check whether this is the case is via *hypothesis testing*.

**Data fits the Gumbel distribution.** We start with the hypothesis that our execution time observations fit the Gumbel distribution:  $\mathcal{H}_0 : F = F_\xi$ . To that end we use the ET test [14] as introduced in Section II-A.

When applying the ET test, we check a relation that is proved equivalent to the hypothesis  $\mathcal{H}_0$  in [14]. This relation verifies that a certain parameter  $\hat{q}_{ET,n}$  calculable for any set

Table I  
P-VALUE FOR THE IDENTICAL DISTRIBUTION TEST FOR DIFFERENT VALUES OF  $m$  FOR A SAMPLE OF  $N=10,000$  OBSERVATIONS. P-VALUE FOR THE INDEPENDENCE TEST

	AT	AI	CA	CN	PU	RS	TB	TT
$m$	Identical distribution test (p-value)							
100	0.34	0.11	0.13	0.41	0.99	0.67	0.77	0.89
500	0.36	0.81	0.24	0.62	0.55	0.84	0.21	0.57
1000	0.16	0.75	0.91	0.88	0.71	0.16	0.92	0.36
$N$	Independence test (p-value)							
10,000	0.66	0.81	0.92	0.84	0.69	0.73	0.76	0.81

of data belongs to an imposed interval. This interval is also proved calculable for any set of data. For all our data we observed that  $\hat{q}_{ET,n}$  belongs to the expected interval.

**Independent and identically distributed observations.** The application of EVT also requires that the data are i.i.d., which is ensured by a combination of hardware with suitable randomisation properties, as discussed in [7], and a suitable experimental methodology when observing the executions of the program under consideration (see Section III).

We start by checking that the data are identically distributed. We apply the two-sample Kolmogorov-Smirnov [28] test in which any two sample sets are compared. The Kolmogorov-Smirnov (KS) statistic quantifies a distance between the empirical distribution functions of two samples. The *null hypothesis*  $\mathcal{H}_0$  is that the both samples are identically distributed and the *alternative hypothesis*  $\mathcal{H}_1$  is that the samples are not identically distributed. For our experiments we use a *significance level*  $\alpha = 0.05$ , which is a common choice in this type of test. The outcome of the test is called the *p-value*. If the p-value is higher than  $\alpha$  the null hypothesis cannot be rejected, which means that both samples are identically distributed.

Table I shows the p-value obtained by applying the KS test to the execution times obtained for a sample of 10,000 observations. From this *original sample* we create two *smaller samples* of  $m$  elements. For this experiment we use three values for  $m$ : 100, 500 and 1000. We populate the smaller samples by randomly taking elements from the original sample, which ensures that the smaller samples maintain the same statistical properties as the original [28]. In our case, the property of interest is the distribution. Then we apply the two-sample KS test to the two smaller samples. In Table I we observe that in all cases the p-value is higher than 0.05 which indicates that data are identically distributed.

In order to prove that samples are independent we use the *runs test* for randomness [5]. This test is used to check whether a series of binary events can be considered to be randomly distributed, and hence independent. In this test, the null hypothesis is that the data are randomly distributed (independent) and the alternative hypothesis is that the data are not randomly distributed (not independent).

A run is defined as a sequence of identical events, preceded or succeeded by different events or no events. The runs test used here applies to binomial distributions only. For example, in a sequence 0110111, we have 4 runs (0, 11, 0, 111). Our

Table II  
MINIMUM NUMBER OF OBSERVATIONS PER EEMBC AUTOBENCH BENCHMARK

AT	AI	CA	CN	PU	RS	TB	TT
300	650	400	450	500	300	500	300

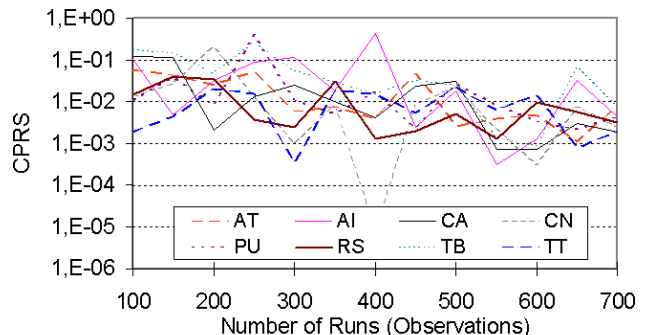


Figure 1. Benchmark CRPS variation as we increase number of runs.  $N_{delta} = 50$  and threshold= 0.1

data are continuous, a cut-point must be chosen by the user so that the data are transformed into a binary sample. The average is chosen as our cut-point. If the execution time is lower than the average, we obtain 0 and in the opposite case 1.

The expectation of the number of runs  $R$  is given by:  $E(R) = 2mn/N$ , where  $m$  is the number of events of type 1, and  $n$  the number of events of type 2, and  $N$  is the total sample size. The variance of the number of runs  $R$  is given by  $V(R) = \frac{2mn \times (2mn - N)}{N^2 \times (N - 1)}$ .

Let  $r$  be the number of runs measured in the sample. It is known that when  $m$  or  $n$  tend to infinity then  $Z = \frac{r - E(R)}{\sqrt{V(R)}} \rightarrow N(0, 1)$ , where  $N(0, 1)$  is the standard normal distribution [5]. This means that the test statistic,  $Z$ , is asymptotically normally distributed. From the value of  $Z$ , we compute the p-value with a significance level  $\alpha = 5\%$ . If the p-value is higher than alpha the null hypothesis cannot be rejected, which means that data are randomly distributed.

The last row in Table I shows the p-value computed for each EEMBC benchmark with 10,000 runs. We observe that for all the benchmarks under consideration the runs we collect from them are independent.

**Minimum Number of Observations** Figure 1 shows the variation of CRPS as we add more runs using  $N_{delta} = 50$ .

In order to obtain the minimum number of observations we use the algorithm presented in Section III-D, which focuses on the convergence of the EVT tail projection as more runs are considered, taking CRPS as the convergence metric. We set the threshold to 0.1 and  $N_{delta}$  to 50. We observe that although CRPS decreases, it is not monotonically decreasing. For this reason our convergence algorithm takes into account small variations. Table II shows the minimum number of runs required for each EEMBC benchmark. On average we need 425 runs per benchmark. The average time required to run an EEMBC benchmark on our simulation infrastructure is about 1 minute, so the total time required

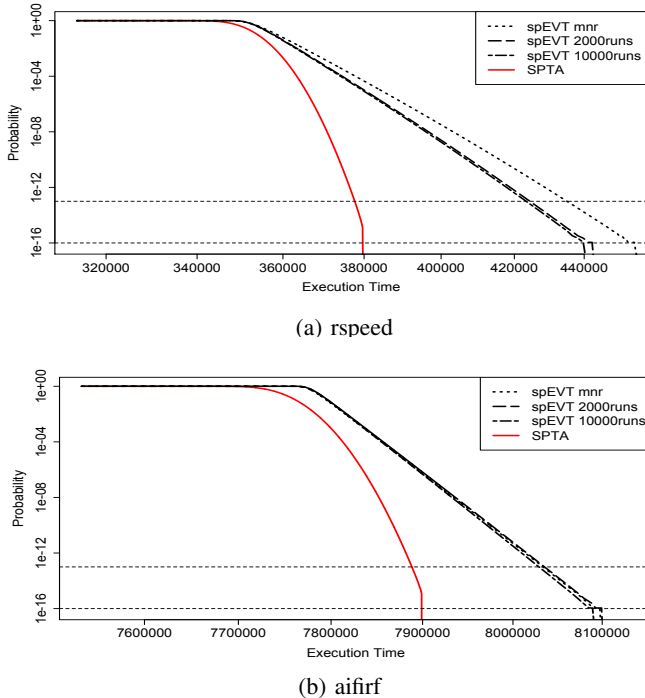


Figure 2. Single-path pWCET estimate for two benchmarks

was  $425 \times 1 \times 8/60 = 57$  hours. This is equivalent to less than one hour of simulations on a 64-node cluster, which is a fairly common asset in industrial production systems. It is worth noting that for building Figure 1 we used 700 runs, which provides satisfactory convergence for a threshold of  $10^{-1}$ . However, to achieve convergence at lower CRPS values, e.g.  $10^{-3}$  we would need to increase the number of observations.

**pWCET estimates.** Figure 2 shows the pWCET estimates provided by MBPTA for two EEMBC benchmarks, *rspeed* and *aifirf* (due to space constraints we cannot show pWCET estimates graphically for all benchmarks). In Figure 2 the x-axis shows the pWCET estimate and the y-axis shows the associated probabilities.

For this experiment we use a processor setup with instruction cache only. The reason for this choice is that the application of SPTA requires the computation of execution time profiles that are hard to obtain for the setup with instruction and data caches.

SPTA and EVT MBPTA can be projected to arbitrarily low probabilities. For the sake of illustration, we set our range of probabilities of interest to lie in the interval  $[10^{-13}, 10^{-16}]$ . This is based on the observation that for commercial airborne systems at the highest integrity level, DAL-A, the maximum allowed failure rate in a system component is  $10^{-9}$  per hour of operation [2]. The highest frequency at which a task can be released is current systems can be assumed to be at 10 milliseconds ( $10^2$  activations per second). Hence, to translate the cited failure rate requirement into the equivalent of at least  $10^9$  hours of continuous operation (i.e.,  $10^2 \times 60 \times 60 \times$

Table III  
MBPTA pWCET ESTIMATES FOR EEMBC BENCHMARKS W.R.T. SPTA

probability	AI	AT	CA	CN	PU	RS	TB	TT
$10^{-13}$	9%	5%	6%	5%	5%	2%	2%	6%
$10^{-16}$	15%	7%	8%	7%	7%	3%	3%	9%

$10^9 = 3.6 \times 10^{13}$ ), the pWCET of that task should have an exceedance probability in the region of  $10^{-13}$ , which falls in the lower end of our probability range.

We show pWCET estimates for our single-path (sp) EVT technique when using the calculated minimum number of runs (*mnr*), 2000 and 10000 runs. For comparison purposes we show the exact execution time estimate we obtain with SPTA. We observe that, for every WCET value, we can compute its exceedance probability. The probabilistic WCET function is smooth, that is, it does not present abrupt changes. Moreover the EVT curve is an upper-bound to the SPTA curve. The pessimism introduced increases as the probability decreases: for probability  $10^{-13}$  it is 2% and 9% respectively for *aifirf* and *rspeed*, increasing to 3% and 15% respectively for a  $10^{-16}$  probability. We also observe that the pWCET estimates provided by EVT have little variation for numbers of runs higher than the minimum number of runs: observed variations are smaller than 3% for all benchmarks for  $10^{-13}$ .

Table III shows the pWCET estimates provided by our EVT technique with respect to the estimates provided by SPTA for the selected EEMBC Autobench benchmarks. We observe that for the  $10^{-13}$  confidence level, the overestimation is less than 9% and for  $10^{-16}$  it is less than 15% for all benchmarks, which is quite acceptable.

#### D. Results for Multi-Path Programs

**pWCET estimates for the synthetic benchmark.** In order to check the results EVT-MBPTA obtains for multi-path programs we use the synthetic benchmark presented in Section V-A. This is a multi-path program for which we know the worst-case path, which is significantly longer than any other path. As a reference comparison point we take the pWCET estimate we obtain with our single-path technique when applied to the worst-case path. We compare it with the pWCET estimates we obtain when apply the multi-path technique to different sets of paths in which we include the worst-case path. For all the experiments in this section, we also applied the test for independence and identical distribution described in the previous section.

Figure 3 shows the pWCET estimate when varying the number of runs for the worst path and for the non-worst paths. We label each estimate as *xxx-yyy* where *xxx* is the number of runs we consider of non worst paths and *yyy* the number of runs from the worst case path. For instance, 0-10000 is the pWCET estimate when we consider 10000 runs of the worst path and 5000-5000 is the pWCET estimate when we consider 5000 runs of non worst-case paths and 5000 runs of the worst-case path. For every path we make at least the minimum number of observations.

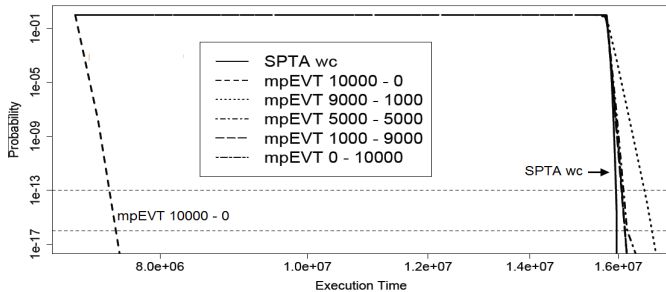


Figure 3. pWCET estimates for the synthetic benchmark

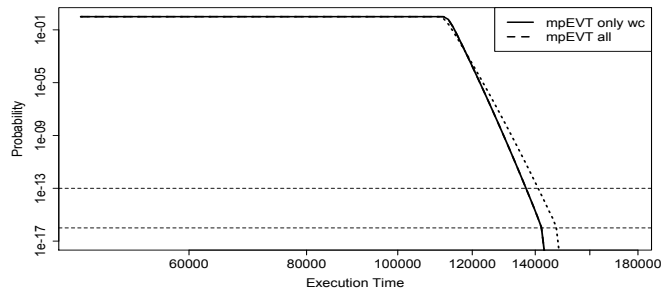


Figure 4. pWCET estimates for the *select* benchmark

Table IV

MBPTA pWCET ESTIMATES FOR THE MALARDALEN BENCHMARKS.  
ONLY WORST-CASE PATH W.R.T. ALL PATHS

probability	BS	CNT	COM	CRC	INS	QSO	SEL
$10^{-13}$	7%	2%	4%	1%	9%	7%	5%
$10^{-16}$	8%	2%	5%	1%	11%	8%	6%

We compare all estimates with the result we obtain with SPTA for the worst-case path (*SPTA wc*). First, and most importantly, we observe that all multi-path estimates that consider the worst path are an upper bound for the SPTA. In the particular case of the 10000-0 estimate, we consider no runs from the worst path in the EVT. This case illustrates our claim in Section IV: pWCET estimates for untested paths cannot be made, as the execution times for those paths would no longer be drawn from an identical distribution. The other EVT estimates that consider the worst path are all an upper bound for the SPTA and have a small variation between them, which indicates that the frequency at which the worst-case path is exercised with the given input data set affects the provided WCET estimate only slightly.

**pWCET estimates for real benchmarks.** We compare here the pWCET estimates we obtain for different Malardalen WCET benchmarks. As in the previous experiments, the data passed all Gumbel fitting and i.i.d tests. We compare the pWCET estimate when we consider 10,000 runs from the worst-case path (wc) against the case in which we consider 10,000 runs from all paths including the worst-case path (all). Figure 4 shows the results for the *select* benchmark. Again we observe that as soon as the worst-case path is considered as part of the data set, EVT provides results comparable to considering only the worst-case path. Table IV shows the pessimism of the latter over the former, which is always less than 11% for the benchmarks we have tested.

## VI. RELATED WORK

The first papers in the real-time community with bearing on our work used the terms *stochastic analysis* [13], *probabilistic analysis* [27], [21], *statistical analysis* [3] and *real-time queuing theory* [20] interchangeably. Since the publication of [8], the notion of *stochastic analysis* of real-time systems has been used regularly by the community, regardless of the approach. In this paper we use the terms *probabilistic analysis*

or *statistical analysis* depending on the approach on which our solution is based. While the former provides the probability or chance of occurrence of future event, the latter searches for a model or some properties when studying some (often large) mass of data of observed past events.

In the real-time domain, as explained in previous sections, earlier proposals have used EVT to provide WCET estimations [18], [9]. The first paper considering the statistical estimation of worst-case execution times is [9], in which extreme value statistics are used to model the WCET. More recently [18] improved the analysis provided in [9] by addressing some flaws in the previous work and produced a refined method also based on extreme value statistics (we develop more on this flaw below). In this paper we cover the problems shown in [16] on the way EVT is applied in the former two papers: the hypothesis of independent and identical distributed random variables and the continuity of the Gumbel distribution function. We also provide a solution to the wrong utilisation of the  $\chi^2$  test in [18] to best fit the Gumbel distribution function, since this test is known not to perform correctly for extreme values.

EVT has been used for solving some different problems in computing systems, ranging from task scheduling [25] to predicting the number of bits software variables need depending on the maximum value they can take, so variable's implementation cost in FPGAs can be reduced [22].

This paper is also the right place to underline that one of the 'mistakes' claimed by [18] in [9] are not really mistakes: [18] notes that [9] does not apply EVT to estimate the WCET, but the execution time of a program. That is, the execution times considered when applying EVT are not the "high" values (worst-case execution times) but all the observed times. To solve this presumed problem [18] introduces the block maxima technique when EVT is applied, so that in each block only the highest observed execution time values are taken. In fact, both approaches are notionally correct, since in [9] the authors use an Exceedance Distribution Threshold Method (EDTM) that selects the largest execution times. Only, EDTM is known to be less robust to small number of observations than EVT with the block maxima method.

## VII. CONCLUSIONS

Probabilistic techniques reduce the cost of acquiring the required knowledge to perform trustworthy WCET analysis, which makes them attractive for industry. In this paper we have presented for the first time a measurement-based PTA technique based on a statistically sound application of EVT which enables the analysis of multi-path programs. We have also identified the statistical properties which hardware and software must demonstrate in order to ensure that probabilistic WCET estimates are sound. We have shown the appropriate tests with which to check whether those properties are present. We have also given a step-by-step description of how to apply our EVT-MBPPTA technique for both single-path and multi-path programs.

Our results show that, for failure probability levels smaller than required in the highest software criticality levels in commercial avionics (i.e.  $10^{-9}$  per hour of operation in DAL-A) our EVT-MBPPTA single-path technique provides pWCET estimates less than 15% higher than Static PTA. This is an acceptable overestimation considering that SPTA provides the exact distribution of execution times of the program. For multi-path programs EVT-MBPPTA provides tight pWCET estimates and presents the key feature that it is independent of the frequency in which the paths under consideration are exercised. This significantly reduces the requirements on the user of our technology, while providing a tight upper bound for pWCET estimates.

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